Parent Packet

HAUPPAUGE MATH DEPARTMENT

CCLS

Grade 6

MODULE 5

http://www.hauppauge.k12.ny.us/math

2014 – 2015 School Year

Grade 6 Module 5

Area, Surface Area, and Volume Problems

In this module, students utilize their previous experiences in order to understand and develop formulas for area, volume, and surface area. Students use composition and decomposition to determine the area of triangles, quadrilaterals, and other polygons. Extending skills from Module 3 where they used coordinates and absolute value to find distances between points on a coordinate plane, students determine distance, perimeter, and area on the coordinate plane in real-world contexts. Next in the module comes real-life application of the volume formula where students extend the notion that volume is additive and find the volume of composite solid figures. They apply volume formulas and use their previous experience with solving equations to find missing volumes and missing dimensions. The final topic includes deconstructing the faces of solid figures to determine surface area. To wrap up the module, students apply the surface area formula to real-life contexts and distinguish between the need to find surface area or volume within contextual situations.

Grade 6 • Module 5

Area, Surface Area, and Volume Problems

OVERVIEW

Starting in Grade 1, students compose and decompose plane and solid figures (1.G.A.2). They move to spatial structuring of rectangular arrays in Grade 2 (2.G.A.2) and continually build upon their understanding of arrays to ultimately apply their knowledge to two- and three-dimensional figures in Grade 4 (4.MD.A.3) and Grade 5 (5.MD.C.3, 5.MD.C.5). Students move from building arrays to using arrays to find area and eventually move to decomposing three-dimensional shapes into layers that are arrays of cubes. In this module, students utilize their previous experiences in shape composition and decomposition in order to understand and develop formulas for area, volume, and surface area.

In Topic A, students use composition and decomposition to determine the area of triangles, quadrilaterals, and other polygons. They determine that area is additive. Students learn through exploration that the area of a triangle is exactly half of the area of its corresponding rectangle. In Lesson 1, students discover through composition that the area of a parallelogram is the same as a rectangle. In Lesson 2, students compose rectangles using two copies of a right triangle. They extend their previous knowledge about the area formula for rectangles (4.MD.A.3) to evaluate the area of the rectangle using $A = bh$ and discover through manipulation that the area of a right triangle is exactly half that of its corresponding rectangle. In Lesson 3, students discover that any triangle may be decomposed into right triangles, and in Lesson 4, students further explore all triangles and discover through manipulation that the area of all triangles is exactly half the area of its corresponding rectangle. During this discovery process, students become aware that triangles have altitude, which is the length of the height of the triangle. The altitude is the perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the base. Students understand that any side of the triangle can be a base, but the altitude always determines the base. They move from recognizing right triangles as categories (4.G.A.2) to determining that right triangles are constructed when altitudes are perpendicular and meet the base at one side. Acute triangles are constructed when the altitude is perpendicular and meets within the length of the base, and obtuse triangles are constructed when the altitude is perpendicular and lies outside the length of the base. Students use this information to cut triangular pieces and rearrange them to fit exactly within one half of the corresponding rectangle to determine that the area formula for any triangle can be determined using $A = 12\ bh$. In Lesson 5, students apply their knowledge of the area of a triangular region, where they deconstruct parallelograms, trapezoids, and other quadrilaterals and polygons into triangles or rectangles in order to determine area. They intuitively decompose rectangles to determine the area of polygons. Topic A closes with Lesson 6 where students apply their learning from the topic to find areas of composite figures in real-life contexts, as well as determine the area of missing regions (6.G.A.1).

In Module 3, students used coordinates and absolute value to find distances between points on a coordinate plane (6.NS.C.8). In Topic B, students extend this learning to Lessons 7 and 8 where they find edge lengths of polygons (the distance between two vertices using absolute value) and draw polygons given coordinates (6.G.A.3). From these drawings, students determine the area of polygons on the coordinate plane by composing and decomposing into polygons with known area formulas. In Lesson 9, students further investigate and calculate the area of polygons on the coordinate plane and also calculate the perimeter. They note that finding perimeter is simply finding the sum of the polygon's edge lengths (or finding the sum of the distances between vertices). Topic B concludes with students determining distance, perimeter, and area on the coordinate plane in real-world contexts.

In Grade 5, students recognized volume as an attribute of solid figures. They measured volume by packing right rectangular prisms with unit cubes and found that determining volume was the same as multiplying the edge lengths of the prism (5.MD.C.3, 5.MD.C.4). Students extend this knowledge to Topic C where they continue packing right rectangular prisms with unit cubes; however, this time the right rectangular prism has fractional lengths (6.G.A.2). In Lesson 11, students decompose a one cubic unit prism in order to conceptualize finding the volume of a right rectangular prism with fractional edge lengths using unit cubes. They connect those findings to apply the formula $V = lwh$ and multiply fractional edge lengths (5.NF.B.4). In Lessons 12 and 13, students extend and apply the volume formula to $V = The\ area\ of\ the\ base \times height$ or simply $V = bh$, where b represents the area of the base. In Lesson 12, students explore the bases of right rectangular prisms and find the area of the base first, then multiply by the height. They determine that two formulas can be used to find the volume of a right rectangular prism. In Lesson 13, students apply both formulas to application problems. Topic C concludes with real-life application of the volume formula where students extend the notion that volume is additive (5.MD.C.5c) and find the volume of composite solid figures. They apply volume formulas and use their previous experience with solving equations (6.EE.B.7) to find missing volumes and missing dimensions.

Module 5 concludes with deconstructing the faces of solid figures to determine surface area. Students note the difference between finding the volume of right rectangular prisms and finding the surface area of such prisms. In Lesson 15, students build solid figures using nets. They note which nets compose specific solid figures and also understand when nets cannot compose a solid figure. From this knowledge, students deconstruct solid figures into nets to identify the measurement of the solids' face edges. With this knowledge from Lesson 16, students are prepared to use nets to determine the surface area of solid figures in Lesson 17. They find that adding the areas of each face of the solid will result in a combined surface area. In Lesson 18, students find that each right rectangular prism has a front, a back, a top, a bottom, and two sides. They determine that surface area is obtained by adding the areas of all the faces. They understand that the front and back of the prism have the same surface area, the top and bottom have the same surface area, and the sides have the same surface area. Thus, students develop the formula $SA = 2lw + 2lh + 2wh$ (6.G.A.4). To wrap up the module, students apply the surface area formula to real-life contexts and distinguish between the need to find surface area or volume within contextual situations.

Topic A

Area of Triangles, Quadrilaterals, and Polygons

In Topic A, students discover the area of triangles, quadrilaterals, and other polygons through composition and decomposition. In Lesson 1, students discover through composition that the area of a parallelogram is the same as the area of a rectangle with the same base and height measurements. Students show the area formula for the region bound by a parallelogram by composing into rectangles and determining that the area formula for rectangles and parallelograms is $A = bh$. In Lesson 2, students justify the area formula for a right triangle by viewing the right triangle as part of a rectangle composed of two right triangles. They discover that a right triangle is exactly half of a rectangle, thus proving that the area of a triangle is $\frac{1}{2}bh$.

Students further explore the area formula for all triangles in Lessons 3 and 4. They decompose triangles into right triangles and deconstruct triangles to discover that the area of a triangle is exactly one half the area of a parallelogram. They decompose the region of a trapezoid into two triangles and determine the area. The topic closes with Lesson 6, where students determine the area of composite figures in real-life contextual situations using composition and decomposition of polygons. They determine the area of a missing region using composition and decomposition of polygons.

Topic B

Polygons on the Coordinate Plane

In lesson 8 students draw polygons in the coordinate plane when given coordinates for vertices. They find the area enclosed by a polygon by composing and decomposing, using polygons with known area formulas. They name coordinates that define a polygon with specific properties. In Lesson 9, students find the perimeter of rectilinear figures using coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. They continue to find the area enclosed by a polygon on the coordinate plane by composition and decomposition. The topic concludes with Lesson 10, where students apply their knowledge of distance, perimeter, and area to real-life contextual situations.

Topic C

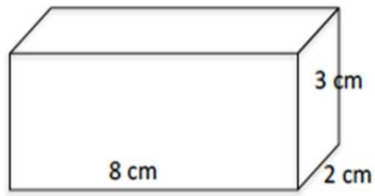
Volume of Right Rectangular Prisms

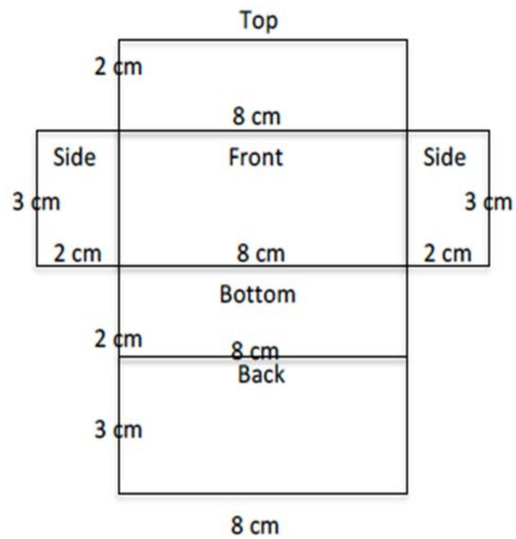
In Topic C, students extend their understanding of the volume of a right rectangular prism with integer side lengths to right rectangular prisms with fractional side lengths. They apply the known volume formula $\boldsymbol{V = lwh}$ to find the the volume of these prisms and use correct volume units when writing the answer.   They determine that two formulas can be used to find the volume of a right rectangular prism. In Lesson 13, students apply both formulas to problems dealing with volume formulas of right rectangular prisms and cubes with fractional edge lengths. The topic concludes with Lesson 14, where students determine the volume of composite solid figures and apply volume formulas to find missing volumes and missing dimensions in real-world contexts.

Topic D

Nets and Surface Area

Topic D begins with students constructing three-dimensional figures through the use of nets in Lesson 15. They determine which nets make specific solid figures and also determine if nets can or cannot make a solid figure. Students use physical models and manipulatives to do actual constructions of three-dimensional figures with the nets. Then, students move to constructing nets of three-dimensional objects using the measurements of a solid's edges. Using this information, students will move from nets to determining the surface area of three-dimensional figures in Lesson 17. In Lesson 18, students determine that a right rectangular prism has six faces: top and bottom, front and back, and two sides. They determine that surface area is obtained by adding the areas of all the faces and develop the formula $SA = 2lw + 2lh + 2wh$. They develop and apply the formula for the surface area of a cube as $SA = 6s^2$.





Top	Bottom	Front	Back	Side	Side
$l \times w$	$l \times w$	$l \times h$	$l \times h$	$w \times h$	$w \times h$
$8\ cm \cdot 2cm$	$8\ cm \cdot 2\ cm$	$8\ cm \cdot 3\ cm$	$8\ cm \cdot 3\ cm$	$2\ cm \cdot 3\ cm$	$2\ cm \cdot 3\ cm$
$16\ cm^2$	$16\ cm^2$	$24\ cm^2$	$24cm^2$	$6\ cm^2$	$6\ cm^2$
$SA = 16\ cm^2 + 16\ cm^2 + 24cm^2 + 24\ cm^2 + 6\ cm^2 + 6\ cm^2 = 92\ cm^2$					
$l \times w$	$l \times w$	$l \times h$	$l \times h$	$w \times h$	$w \times h$
$2lw$		$2lh$		$2wh$	
$SA = 2lw + 2lh + 2wh$					

Topic D concludes with Lesson 19, where students determine the surface area of three-dimensional figures in real-world contexts. To develop skills related to application, students are exposed to contexts that involve both surface area and volume. Students are required to make sense of each context and apply concepts appropriately.

Terminology

New or Recently Introduced Terms

- **Triangular Region** (A *triangular region* is the union of the triangle and its interior.)
- **Altitude and Base of a Triangle** (An *altitude* of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the *base*. For every triangle, there are three choices for the altitude, and hence there are three base-altitude pairs. The *height* of a triangle is the length of the altitude. The length of the base is either called the *base length* or, more commonly, the *base*. Usually, context makes it clear whether the *base* refers to a number or a segment. These terms can mislead students: base suggests the bottom, while *height* usually refers to vertical distances. Do not reinforce these impressions by consistently displaying all triangles with horizontal bases.)
- **Pentagon** (Given 5 different points A, B, C, D, E in the plane, a 5-*sided polygon,* or *pentagon*, is the union of five segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear.)
- **Hexagon** (Given 6 different points A, B, C, D, E, F in the plane, a 6-*sided polygon,* or *hexagon,* is the union of six segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EF}, \overline{FA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear. For both pentagons and hexagons, the segments are called the *sides*, and their endpoints are called the *vertices*. Like quadrilaterals, pentagons and hexagons can be denoted by the order of vertices defining the segments. For example, the pentagon $ABCDE$ has vertices A, B, C, D, E that define the five segments in the definition above. Similar to quadrilaterals, pentagons and hexagons also have *interiors*, which can be described using pictures in elementary school.)

- **Line Perpendicular to a Plane** (A line L intersecting a plane E at a point P is said to be *perpendicular to the plane* E if L is perpendicular to every line that (1) lies in E and (2) passes through the point P. A segment is said to be perpendicular to a plane if the line that contains the segment is perpendicular to the plane. In Grade 6, a line perpendicular to a plane can be described using a picture.)
- **Parallel Planes** (Two planes are *parallel* if they do not intersect. In Euclidean geometry, a useful test for checking whether two planes are parallel is if the planes are different and if there is a line that is perpendicular to both planes.)
- **Right Rectangular Prism** (Let E and E' be two parallel planes. Let B be a rectangular region[4] in the plane E. At each point P of B, consider the segment $\overline{PP'}$ perpendicular to E, joining P to a point P' of the plane E'. The union of all these segments is called a *right rectangular prism.* It can be shown that the region B' in E' corresponding to the region B is also a rectangular region whose sides are equal in length to the corresponding sides of B. The regions B and B' are called the *base faces* (or just *bases*) of the prism. It can also be shown that the planar region between two corresponding sides of the bases is also a rectangular region called the *lateral face* of the prism. In all, the boundary of a right rectangular prism has 6 *faces*: the 2 base faces and 4 lateral faces. All adjacent faces intersect along segments called *edges* (base edges and lateral edges).)
- **Cube** (A *cube* is a right rectangular prism all of whose edges are of equal length.)
- **Surface of a Prism** (The *surface of a prism* is the union of all of its faces (the base faces and lateral faces).)

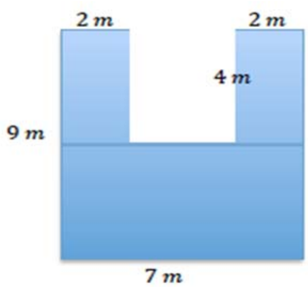
Familiar Terms and Symbols[5]

- Perimeter
- Area
- Volume
- Angle
- Triangle
- Quadrilateral
- Parallelogram
- Trapezoid
- Rectangle
- Square
- Perpendicular
- Parallel
- Segment
- Length of a Segment

Lesson 5

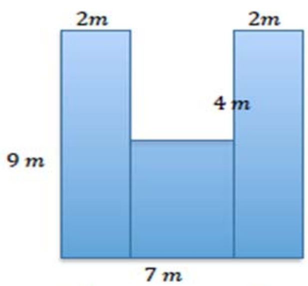
<u>Objective:</u> The Area of Polygons Through Composition and Decomposition

Decomposing Polygons into Rectangles



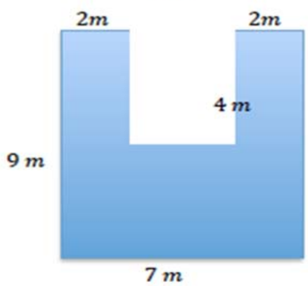
Area of Rectangle 1: $b \cdot h$
$2\ m \cdot 4\ m = 8\ m^2$
Area of Rectangle 2: $b \cdot h$
$2\ m \cdot 4m = 8\ m^2$
Area of Rectangle 3: $b \cdot h$
$7\ m \cdot 5\ m = 35\ m^2$

Area of Polygon: $8\ m^2 + 8\ m^2 + 35\ m^2 = 51\ m^2$



Area of Rectangle 1: $b \cdot h$
$9\ m \cdot 2\ m = 18\ m^2$
Area of Rectangle 2: $b \cdot h$
$9\ m \cdot 2m = 18\ m^2$
Area of Rectangle 3: $b \cdot h$
$3\ m \cdot 5\ m = 15\ m^2$

Area of Polygon: $18\ m^2 + 18\ m^2 + 15\ m^2 = 51\ m^2$



Area of Rectangle: $b \cdot h$
$9\ m \cdot 7\ m = 63\ m^2$
Area of Missing Rectangle 2: $b \cdot h$
$3\ m \cdot 4\ m = 12\ m^2$

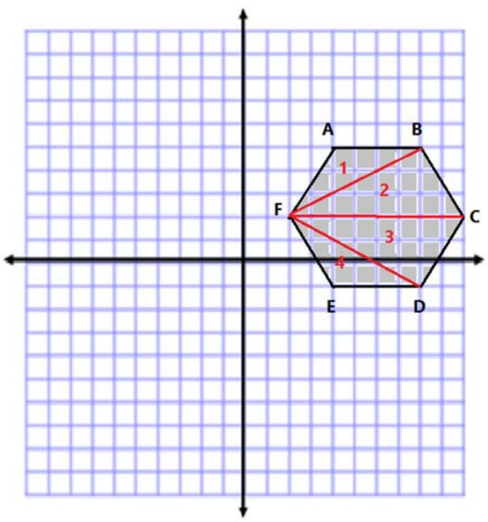
Area of Polygon: $63\ m^2 - 12\ m^2 = 51\ m^2$

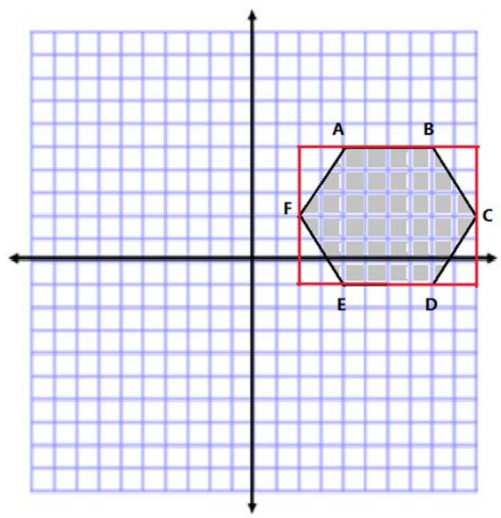
Lesson 9

<u>Objective:</u> Determining Area and Perimeter of Polygons on the Coordinate Plane

<u>Example:</u> Calculate the area of the polygon using two different methods. Write two expressions to represent the two methods and compare the structure of the expressions.

Answers will vary. The following are two possible methods. However, students could also break the shape into two triangles and a rectangle or another correct method.





Area of triangle 1 and 4

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4\ units)(3\ units)$

$A = \frac{1}{2}(12\ units^2)$

$A = 6\ units^2$

Since there are 2, we have a total area of 12 units².

Area of triangle 2 and 3

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(8\ units)(3\ units)$

$A = \frac{1}{2}(24\ units^2)$

$A = 12\ units^2$

Since there are 2, we have a total area of 24 units².

Total area $= 12\ units^2 + 24\ units^2 = 36\ units^2$

$A = lw$

$A = (8\ units)(6\ units)$

$A = 48\ units^2$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2\ units)(3\ units)$

$A = 3\ units^2$

There are 4 triangles of equivalent base and height.

$4(3\ units^2) = 12\ units^2$

Total area $= 48\ units^2 - 12\ units^2$

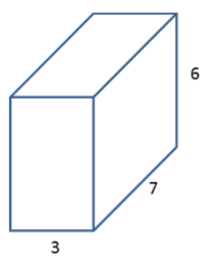
Total area $= 36\ units^2$

Lesson 16

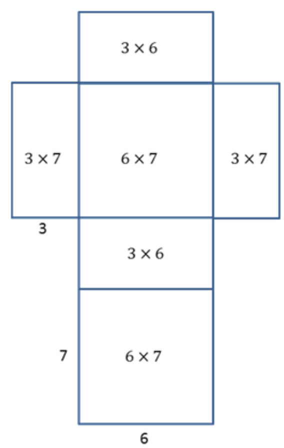
<u>Objective:</u> Constructing Nets

<u>Example:</u> Students will make nets from given measurements. Rectangles should be cut from graph paper and taped. Ask students to have their rectangle arrangements checked before taping. After taping, it can be folded to check its fidelity.

Use the measurements from the solid figures to cut and arrange the faces into a net.



One possible configuration of rectangles is shown here:



Technology Resources

www.k-5mathteachingresources.com -This site provides an extensive collection of free resources, math games, and hands-on math activities aligned with the Common Core State Standards for Mathematics.

www.parccgames.com – fun games to help kids master the common core standards.

http://www.mathplayground.com –common core educational math games and videos.

www.learnzillion.com – math video tutorials.

www.ixl.com – practice common core interactive math skills practice.

www.mathnook.com –common core interactive math skill practice/ games, worksheets and tutorials.

www.adaptedmind.com – common core interactive practice, video lessons and worksheets

www.brainpop.com – animated tutorials of curriculum content that engages students. Can use a limited free version or buy a subscription.